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ABSTRACT

This paper focuses specifically on the reasoning skill called proportional reasoning. Areas addressed include: (1) a definition (providing examples of direct and indirect proportional reasoning); (2) the development of proportional reasoning (discussing survey results of students logical reasoning abilities); (3) planning for the future (proposing steps for increasing students' understanding of proportional reasoning and analyzing the teaching approaches used in other countries where the development of proportional reasoning has been more successful and convey this information to U.S. teacher training institutions); and (4) a mental processing model for proportional reasoning (presenting a model which identifies a hierarchy of skills and mental processes needed for proportional reasoning). A list of 18 references is included. (ML)

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TOWARD THE ADVANCEMENT OF TEACHING LOGICAL THINKING: A MENTAL PROCESSING MODEL FOR PROPORTIONAL REASONING

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INTRODUCTION

The purpose of this paper is to focus on teaching students to think logically. With both a general audience and a teacher-researcher audience in mind, it is hoped that this paper will advance the development of students' logical thinking by promoting both public and professional understanding of some of the challenges associated with teaching logical thinking, especially when it involves diverse student populations, and by promoting discussion of issues related to this task as some student populations become increasingly diverse in their prior knowledge and previous experience.

Logical reasoning is narrowed to consider a specific reasoning skill called proportional reasoning. Examples of proportional reasoning are used to illustrate the importance of this skill in everyday life. An experienced-based analysis of proportional reasoning and its prerequisite skills is offered so that the general reader may develop an appreciation for the challenges faced by teachers who teach proportional reasoning to their students, and an appreciation for the importance of assumptions that teachers make about their students' prior knowledge and previous experience.

A second, theoretically derived analysis of proportional reasoning is presented to encourage teachers and researchers to reconsider their assumptions about essential prerequisite skills of proportional reasoning, and to look for stumbling blocks which may previously have prevented some students from acquiring this reasoning skill in spite of thoughtful and careful instruction.

DEFINITION

What is proportional reasoning? It is easiest to illustrate proportional reasoning through commonplace examples. Consider the following example. Whenever someone goes to the store to purchase a product and compares the cost of two different sizes of the same product, the task involves comparing two ratios to decide which size of packaging is most economical (e.g., $2.40/6 oz. or $2.80/for 8 oz.). Surveys have shown that this proportional reasoning task can be completed by 33% or less of all supermarket shoppers surveyed (Capon and Kuhn, 1979).

A further example of the need for proportional reasoning occurs whenever someone travels in a foreign country and must convert his own national currency into foreign currency. The task of comparing the cost of a woolen scarf in Norwegian kroner or British pounds with one's own national currency in Japanese yen or American dollars requires the comparison of two ratios: $1.00/6.24 k (the June 1986 exchange rate between U.S. dollars and Norwegian kroner) and x/250k (the unknown cost of the scarf in U.S. dollars compared with the cost of the scarf in Norwegian kroner).

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Both of these examples illustrate situations where an increase in one factor is accompanied by a simultaneous increase in another factor, or the reverse, where simultaneous decreases occur. For example, as the size of a supermarket product increases, the price also increases -- although usually not at the same rate. What does not happen is an increase in the size with a simultaneous decrease in cost (e.g., $2.40 for 6 oz. and $2.00 for 8 oz.). Both of these examples then illustrate a kind of proportional reasoning called DIRECT proportional reasoning where there is a change (increase or decrease) in the same direction found in both factors of a second ratio compared with the first ratio.

There is a second kind of proportional reasoning that is INDIRECT which is used to describe situations where two factors change in opposite directions. That is to say, when one factor increases, the other decreases. To illustrate, when two people of unequal weights sit on a teeter-totter and attempt to balance their weights, the heavier person must sit nearer the center of the teeter-totter (i.e., the fulcrum) while the lighter person sits nearer the end of the opposite side. In this example, as weight increases (the weight of one person in comparison to another), the distance from the fulcrum or the center of the teeter-totter decreases (in comparison to the other person's distance from the fulcrum).

Another example of INDIRECT proportionality is seen in the relationship of weight to exercise. The more exercise an overweight person gets, the more weight he or she loses; and conversely, the less exercise one gets, generally the more weight one gains (assuming the food intake remains the same).

To define proportional reasoning then in terms of skills to be developed, one must generally be able to recognize situations in which two factors acting together as a ratio, either DIRECTLY or INDIRECTLY, are related or equivalent to two other factors which act together as a ratio. Situations which require proportional reasoning frequently have four factors already given (e.g., two sizes of a product and two prices of the same product) and one must find out if the two sets of factors are equivalent. Just as often, however, only three of four factors are known and one must solve for the "unknown" as the example of the teeter-totter illustrates. The "knowns" are the approximate weights of the two people and the fact that the less heavy person must sit at the very end of the teeter-totter; the "unknown" is where the heavier person should sit on the opposite side. A parallel example of the teeter-totter is found in the balance beam which is a piece of equipment used in science labs to compare weights of objects or materials.

Until now, proportional reasoning has been discussed through examples that are commonplace. While proportional reasoning is a desirable skill for these and many other common experiences of the kind just described, it is absolutely esential for understanding concepts of major importance in the various fields of science and mathematics, both at the high school and college or university levels. In order to balance equations that characterize chemical reactions like photosynthesis or electrolysis, for example, or to solve problems which require the comparison of two different ratios such as speed/mass or pressure/temperature, students must be capable of both DIRECT and INDIRECT proportional reasoning.

Yet American teachers and researchers alike report having great difficulty when teaching proportional reasoning to heterogeneous groups of students. In spite of their best efforts, many teachers are unsuccessful with about 1/3 of their students who seem unable to master or understand proportional reasoning. One serious consequence of this fact is the loss of opportunity for students who are thus unable to pursue careers in math or science, and the likelihood that they will feel unsuccessful even in their general math and science courses.

THE DEVELOPMENT OF PROPORTIONAL REASONING

What do we know about the development of proportional reasoning? About thirty years ago Jean Piaget, the Swiss epistomologist and psychologist, and his colleague, Barbel Inhelder, described with great care and detail the results of their study of the development of logical thought or reasoning from childhood to adolescence (1958). They considered proportional reasoning an essential element for logical thought.

Shortly after translations of the research of Piaget and Inhelder were published, researchers throughout the world began to repeat their experiments, eager to verify the existence of identifiable stages in the development of logical thought which Piaget and Inhelder had described. Most researchers used the same clinical interview technique used by the Swiss team to assess logical reasoning abilities of children and adolescents. In the clinical interview, students are presented with materials (e.g., the balance beam mentioned earlier) which can be manipulated to solve problems. A few researchers began to experiment with paper-and-pencil tests of reasoning abilities, in an effort to gather information more quickly with large groups of students than they were able to gather with the one-to-one clinical interviews.

The results of this flurry of worldwide surveying of students' logical reasoning abilities were mixed. An invariant sequence of development seemed to be verified on all sides; but significant differences in reasoning abilities across nations and student groups began to emerge. American researchers were startled to find that the majority of U.S students (about 75%) did not develop the capability for proportional reasoning at the ages reported by Piaget and Inhelder among Swiss students (i.e., ages 11-12 years). U.S. researchers were further dismayed to find that less than half of the American students surveyed were able to use proportional reasoning during their junior and senior years of high school. Table 1: International Survey of Proportional Reasoning Performance of Students Aged 11-18 Years provides an overview of results from surveys in 1978.

In their attempts to understand why proportional reasoning was so difficult for U.S. students, many American researchers developed a variety of materials and techniques for teaching proportional reasoning to high school and university students (Karplus and Peterson, 1970, 1972, 1976; Kurtz, 1976; Kurtz and Karplus, R., 1977; Lawson, 1976, 1977, 1978; Peterson and Streitberger, 1978; Renner and Paske, 1977; Wollman and Lawson, 1978).

Table 1: International Survey of Proportional Reasoning Performance of Students Aged 11-18 Years*

Population Sample	Ratio Approach		Population Sample	Ratio Approach	
	Item 1	Items 1 and 2		Item 1	Items 1 and 2
China[a]			Denmark[a]		
Age 11-12			Age 14		
4th-5th Grade	92	76	7th Grade	37	16
United States[a]			Sweden[a]		
Age 11-12			Age 14-15		
6th Grade			7th-8th Grades		
Upper Middle	21	—	Middle Class	54	29
			Working Class	25	11
United States[a]					
Age 13-14			Austria[b]		
8th Grade			Age 14-15		
Upper Middle	34	20	8th Grade		
Middle Class	25	15	Top Group	74	49
Urban Low Income	2	0	Middle Group	52	33
			Low Group	35	16
United States[a]					
Age 16-18			Germany[b]		
11th-12 Grades			Age 15		
Upper Middle	75	53	9th Grade		
Middle Class	48	31	Top Group	89	64
Urban Low Income	16	11	Middle Group	66	39
			Low Group	41	7
Italy[a]					
Age 13-14			Great Britain[b]		
8th Grade			Age 14		
Upper Middle	70	40	8th Grade		
Middle Class	43	25	Top Group	90	70
Working Class	31	18	Middle Group	70	45
			Low Group	29	13

a: Comprehensive School System
b: Selective School System

*The data in this table are provided by Robert Karplus, and first appeared in The Genetic Epistomologist 7 no. 3 July, 1978.

Recently Lawson reviewed over 150 studies which have tested Piaget's and Inhelder's theory about the development of logical thought and included many studies of reasoning skills other than proportional reasoning (1985). One is struck by two things in Lawson's review: the small number of investigations of proportional reasoning ability, and the small gains that seem to have been made in understanding how better to teach such an important skill. This observation is not a criticism of either Lawson's review or the studies themselves but rather a commentary on the challenge still to be met. From Lawson's review, one learns:

- That the development of proportional reasoning appears to be sequential, and the sequence appears to be hierarchical.
- That teaching which follows an order of presentation from simple to complex tasks of proportional reasoning is more effective than other orders of presentation.
- That active (called hands-on) learning is more effective for teaching proportional reasoning than textbook learning for students in junior high school, but that both hands-on and textbook methods of learning seem equally effective for high school students.

Yet nearly 30 years after Piaget and Inhelder first published their results, American teachers or researchers do not know why less than 25% of sixth grade students from upper middle income levels have developed proportional reasoning skills -- or of even greater concern -- why less than 20% of economically disadvantaged students from urban schools appear to develop proportional reasoning skills by the eleventh or twelfth grades. The concern is for the lost opportunities represented by the lack of this reasoning skill. When one compares the performance of U.S. eighth grade students with their peers in other countries (see Table 1), it begins to look like differences in performance, regardless of socio-economic levels of students, are related to differences in instruction.

PLANNING FOR THE FUTURE

Where do we go from here? If the present situation in the U.S. is to be improved, two steps need to occur. First, American researchers need to increase and coordinate their efforts to identify the stumbling blocks that stand in the way of U.S. students' understanding of proportional reasoning; and second, teachers and researchers need to analyze the teaching approaches used in other countries where the development of proportional reasoning has been more successful, and convey this information to U.S. teacher training institutions.

An important first step then is the development of a model that characterizes the mental processes used by successful proportional reasoners as they solve problems involving the comparison of ratios. Such a mental processing model must be empirically tested and refined. Then it can be useful both as a diagnostic tool to identify the stumbling blocks encountered by students who are unsuccessful in proportional reasoning, and also can be useful as an instructional procedure.

What follows shortly is a proposed mental processing model which presents in hierarchical form and sequence many mental processes that are involved in solving problems which require either DIRECT or INDIRECT proportional reasoning. The format of this mental processing model is styled along lines and procedures suggested by Gagne (1970), Larkin (1979), and Simon and Simon (1979). Larkin has suggested that information processing models illustrate the skills and capabilities that an expert possesses, and show how they differ from the skills and capabilities of beginners. To develop such models, Larkin proceeds in the following way:

1. By making detailed observations of how a few individuals perform the tasks of interest. Commonly this is done by asking the person to "think aloud" while performing the task, and recording his comments for future analysis by means of an audio or video record.

2. On the basis of these detailed data, a model of the mental processes involved in the task is constructed. This model is written as a "program" which can be implemented by another person following its directions, and which actually produces samples of behavior relevant to the problem or task to be solved.

3. The observable output produced by the model is compared with the work of other persons performing the same task (Larkin, 1979, pp. 109-110).

The content of the mental processing model presented in the next section is based on original descriptions of the development of proportional reasoning from childhood to adolescence by Piaget and Inhelder (1958) and upon original research performed by the author and colleagues (Karplus and Peterson, 1970, 1972, 1976; Peterson and Streitberger, 1978). This content concerns two tasks or problems that traditionally have been used by researchers to assess students' levels of proportional reasoning ability. The tasks are referred to as (1) the Mr. Short, Mr. Tall problem, and (2) the Balance Beam problem. The Mr. Short, Mr. Tall problem was originally described by Karplus and Peterson (1970) but here is presented in a modified form by Fuller; it is a task of DIRECT proportional reasoning ability. The Balance Beam problem was first described by Piaget and Inhelder (1958) and is used to assess INDIRECT proportional reasoning ability. Both of these tasks are presented here in their paper-and-pencil format, but they are often presented to students in a hands-on format where actual materials and equipment are used.

[Insert Figure 1 about here.]

Figure 1: Mr. Short, Mr. Tall Problem

Mr. Short is shown below. Mr. Tall is *not* shown.

Mr. Short's height has been measured with large buttons one above the other. Four buttons reached from the floor to the top of his head. Thus:

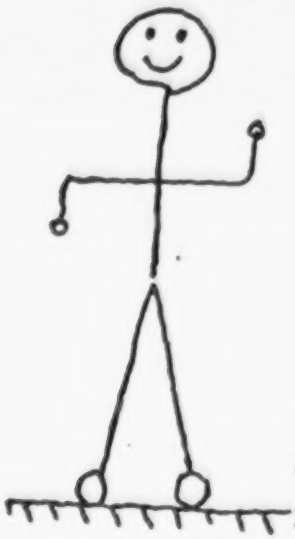
Mr. Short is 4 buttons high.

Mr. Tall measured with the same buttons was 6 buttons high. Thus:

Mr. Tall is 6 buttons high.

Mr. Short, as shown, has also been measured with paper clips. We find:

Mr. Short is 6 paper clips high.



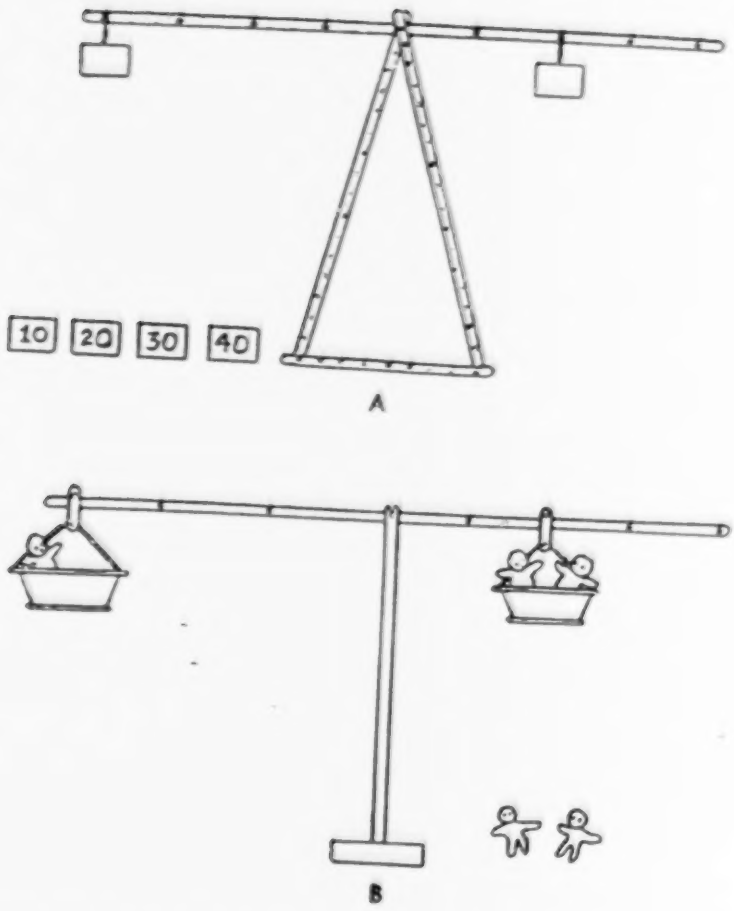
Part A. What is the height of Mr. Tall, measured in paper clips? _____

Please explain carefully how you found this answer.

Part B. Mr. Tall's car is 14 paper clips wide. How wide is Mr. Tall's car, measured in buttons? _____

Please explain carefully how you found this answer.

Figure 2: The Balance Beam Problem



The balance scale is here shown in two forms: (A) a conventional balance with varying weights which can be hung at different points along the crossbar; (B) a balance equipped with baskets which can be moved along the crossbar to different points and in which dolls are used as weights.

Students are presented with the situation where 2 weights are hung at unequal distances, creating an imbalance and asked, "Can you make it (the beam) straight (balanced)? Other arrangements of weights and distances are presented to the student and he/she is asked to balance the beam. If the student is able to balance the beam, he/she is asked to explain how it (the balance beam) works.

Source: Piaget, J. and B. Inhelder, The Growth of Logical Thinking from Childhood to Adolescence. Basic Book, Inc. 1958.

A MENTAL PROCESSING MODEL FOR PROPORTIONAL REASONING

What mental processes must a student command in order to solve these two problems requiring proportional reasoning? At the most advanced level, students are comparing two ratios. In the DIRECT proportional reasoning task, they are comparing:

$$\frac{\text{Mr. Short's height in buttons}}{\text{Mr. Tall's height in buttons}} \text{ to } \frac{\text{Mr. Short's height in paper clips}}{\text{Mr. Tall's height in paper clips}}$$

At the computational level they have to know that:

$$\frac{4 \text{ buttons}}{6 \text{ buttons}} \text{ is compared with } \frac{6 \text{ paper clips}}{x \text{ paper clips}}$$

and thus $4x = 36$ and
$x = 9$ paper clips, Mr. Tall's height

But the opportunity to compare or control four numbers in two ratios is a complex skill. Comparing two ratios is very much like juggling four balls. Anyone who has tried to juggle four balls knows that the first step is to master throwing and catching one ball, then two balls, adding a third when two are mastered, and finally adding the fourth ball. It is a process of gradual expansion of the skill of throwing and catching one ball.

The metaphor of juggling balls is useful for thinking about proportional reasoning because it illustrates an important finding by Piaget and Inhelder (1958) which has been replicated by numerous other researchers who have studied the development of logical reasoning. It has been found that young children (ages 5-7 years) can focus on only one factor at a time. For example, when they are asked to compare two clay balls of equal volume before and after one has been rolled into a sausage, they consistently conclude that the sausage-shaped ball is bigger than the round ball. In this task they focus on the length of the sausage, to the exclusion of other dimensions of the two clay balls. In numerous tasks or problems presented to children of this age, similar results show their capacity to focus on two or more factors has not yet developed.

In contrast, during their middle years (ages 7-11, 12 years) children develop the capacity to focus on several factors simultaneously and are able to compare factors like height and width, distance and speed, or volume and weight; but they still have not developed the ability to explain rules that govern the relationship between or among factors or to mentally manipulate such variables. Only in the adolescent years are students able to see or test and describe these relationships and manipulations.

Given the complexity of proportional reasoning, an attempt has been made to describe here some of the prerequisite skills that may be inherent in proportional reasoning for the Mr. Short, Mr. Tall problem and the Balance Beam problem.

Figure 3 is an experienced-based analysis of proportional reasoning for the two tasks presented above. By experience-based is meant that the content of the mental processing model represents both intuitive responses (e.g., hunches, dimly seeing "the big picture") and computational reasoning skills which have been expressed by students in the process of solving the problems.

On the left side of Figure 3 are statements or calculations representing skills associated with the DIRECT proportional reasoning task, Mr. Short, Mr. Tall. On the right side are reasoning skills associated with the Balance Beam problem, and thought to be at the same levels of development as their counterparts on the left.

The reasoning skills in Figure 3 are organized to suggest a hierarchy with simpler skills placed at the bottom of the page to imply not only their early acquisition in a developmental sequence but also that each level of skill is prerequisite to the level above it.

One can begin to imagine the challenge faced by teachers who want to teach proportional reasoning to their students if the skills identified in Figure 3 represent prerequisites. If so, then teachers must surely check to see how many of the students in the class possess the more elementary reasoning skills before teaching them more advanced thinking skills.

Numbers appear on the left side of the page in Figure 3 to facilitate discussion of specific reasoning skills and to make easier the comparison of specific levels of reasoning in Figure 3 with their counterparts in Figure 4.

Figure 4 is a theoretically derived analysis of proportional reasoning based on the original research of Piaget and Inhelder (1958) and others mentioned earlier (Karplus and Peterson, 1970, 1972, 1976; Peterson and Streitberger, 1978). The mental processing model in Figure 4 represents an attempt to show how at least two other reasoning skills described by Piaget and Inhelder are part of the total mental structure or SCHEMA for proportional reasoning. These include developing the ability to recognize and manipulate problems involving one-to-one (1:1) correspondence, and similarly, problems involving serial order (e.g., 2, 4, 6, 8, 10...n).

In one important way, Figures 3 and 4 differ from other mental processing models. The skills described at the middle levels in boxes 5, 6 and 7 are not based on observed responses of students solving the problems, but rather they are hypothesized skills. Why? Very simply, to fill an "unknown," a gap in observations. In the past, students who solved proportional reasoning have been observed to exhibit the skills at the top levels in the model while students who tried and failed to solve these proportional reasoning problems have exhibited the skills at the bottom levels but never the top levels. The fact that so many students in certain groups (30%-80%) could not solve proportional reasoning problems suggested that some yet-to-be-recognized skills were crucial but missing in the process and not routinely provided in instruction. Operating on the assumption that a gap existed in what was assumed to be a developmental sequence, an effort was made to conceptualize what the missing skills or processes might be. They are described next.

It is hypothesized that the ability to recognize and solve problems involving one-to-many (1:MANY) correspondence (e.g., one teacher to thirty students, or one button to 1-1/2 paper clips) is a prerequisite skill for proportional reasoning. Likewise, it is hypothesized that students must be able to serial order sets of numbers representing one-to-many correspondences. For example, 1 button:1-1/2 paper clips, 2 buttons:3 paper clips, 3 buttons:4-1/2 paper clips, and 4 buttons:6 paper clips. Both of these hypothesized reasoning skills are presented in the hierarchical arrangement of mental processes shown in Figures 3 and 4. To the best of the author's knowledge, neither of these hypothesized reasoning skills has been recognized before as prerequisite to proportional reasoning.

Finally, Figure 4 attempts to integrate into a mental processing model four generic processes, INRC, which have been described by Piaget and Inhelder (1958) but which rarely appear in instructional settings or materials that are intended to develop logical thinking. The INRC group of mental processes refer to:

- initiating an action (I)
- negating the action, undoing the action (N)
- forming a reciprocal of the initial action such that it counteracts or balances the first action (R)
- canceling the reciprocal action (C)

This group of mental processes is best illustrated by the actions taken to establish equilibrium on the Balance Beam problem, but they (the INRC group of mental processes) are important throughout the development of logical thinking skills. In the Balance Beam problem:

I can be seen as the initial placement of a weight on one side of the fulcrum. A condition of disequilibrium is created by I.

N can be seen as removing the weight (negating the initial action) thus restoring equilibrium to the empty balance beam.

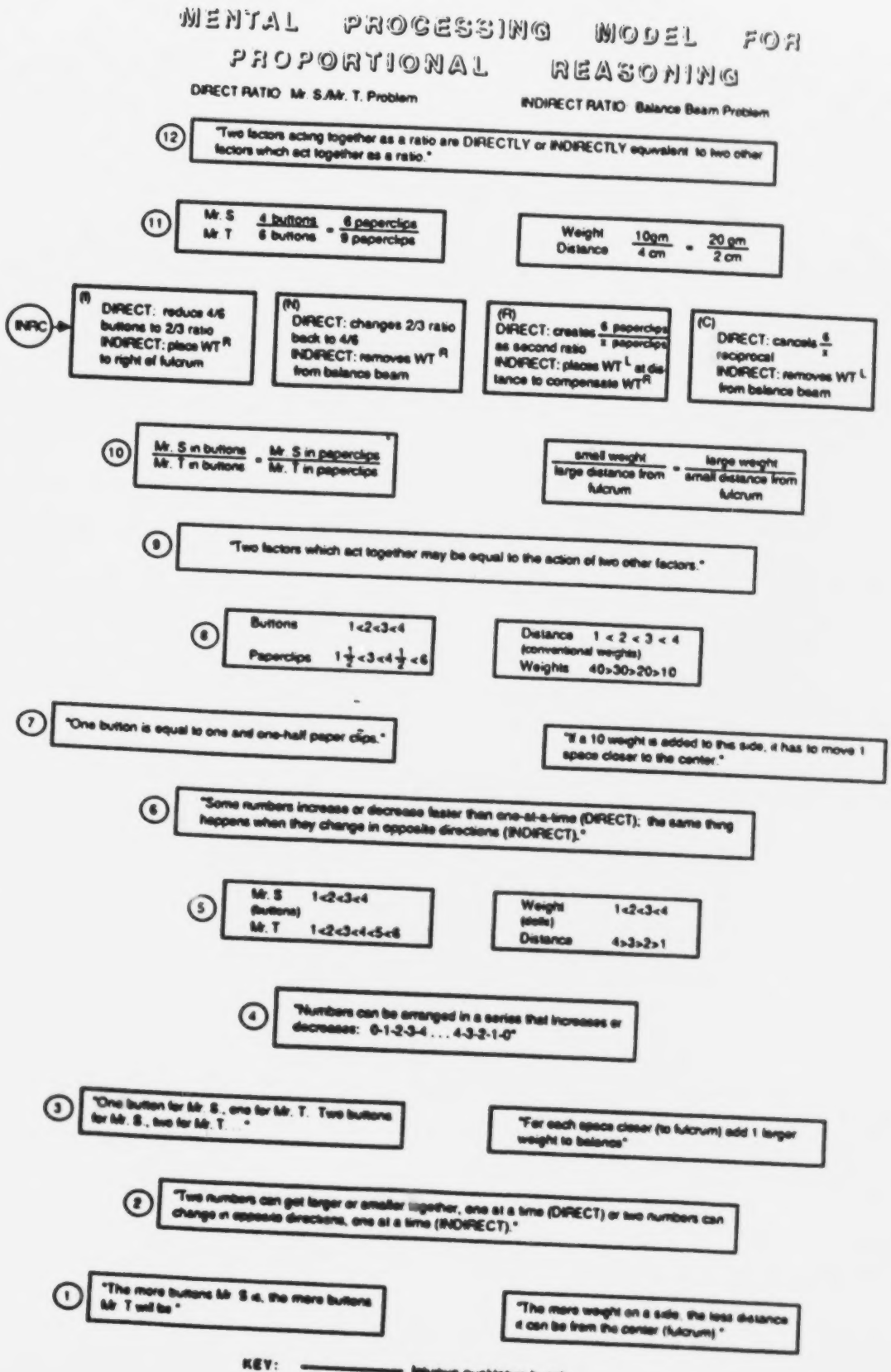
R can be seen as adding an equal weight to the other side of the balance beam, the same distance from the fulcrum as the first weight, thus balancing the beam (assuming the initial weight is still there).

C can be seen as removing the second weight, leaving only the first weight in place, and thus restoring the balance to disequilibrium, a state the initial I action created.

Obviously INRC processes are used in solving the Mr. Short, Mr. Tall problem as well as the Balance Beam problem. The significance of the INRC processes is that they enable students to try various actions and reverse them, giving a sense of mental security or equilibrium while solving problems.

[Insert Figures 3 and 4 about here.]

Figure 3


MENTAL PROCESSING MODEL FOR PROPORTIONAL REASONING
DIRECT RATIO Mr. S./Mr. T. Problem
INDIRECT RATIO Balance Beam Problem
(12) "Two factors acting together as a ratio are DIRECTLY or INDIRECTLY equivalent to two other factors which act together as a ratio."
(11) Mr. S / Mr. T: 4 buttons / 6 buttons = 6 paperclips / 9 paperclips
Weight / Distance: 10gm / 4 cm = 20 gm / 2 cm
INRC
(I) DIRECT: reduce 4/6 buttons to 2/3 ratio INDIRECT: place WT R to right of fulcrum
(N) DIRECT: changes 2/3 ratio back to 4/6 INDIRECT: removes WT R from balance beam
(R) DIRECT: creates 6 paperclips / x paperclips as second ratio INDIRECT: places WT L at distance to compensate WT R
(C) DIRECT: cancels 6/x reciprocal INDIRECT: removes WT L from balance beam
(10) Mr. S in buttons / Mr. T in buttons = Mr. S in paperclips / Mr. T in paperclips
small weight / large distance from fulcrum = large weight / small distance from fulcrum
(9) "Two factors which act together may be equal to the action of two other factors."
(8) Buttons 1<2<3<4
Paperclips 1 1/2 < 3 < 4 1/2 < 6
Distance 1 < 2 < 3 < 4 (conventional weights)
Weights 40>30>20>10
(7) "One button is equal to one and one-half paper clips."
"If a 10 weight is added to this side, it has to move 1 space closer to the center."
(6) "Some numbers increase or decrease faster than one-at-a-time (DIRECT); the same thing happens when they change in opposite directions (INDIRECT)."
(5) Mr. S (buttons) 1<2<3<4
Mr. T 1<2<3<4<5<6
Weight (dots) 1<2<3<4
Distance 4>3>2>1
(4) "Numbers can be arranged in a series that increases or decreases: 0-1-2-3-4 . . . 4-3-2-1-0"
(3) "One button for Mr. S., one for Mr. T. Two buttons for Mr. S., two for Mr. T. "
"For each space closer (to fulcrum) add 1 larger weight to balance"
(2) "Two numbers can get larger or smaller together, one at a time (DIRECT) or two numbers can change in opposite directions, one at a time (INDIRECT)."
(1) "The more buttons Mr. S. . the more buttons Mr. T will be."
"The more weight on a side, the less distance it can be from the center (fulcrum)."
KEY: Intuitive qualitative hunch
Action followed by stated conclusion

RWP: 86

Figure 4

MENTAL PROCESSING MODEL FOR PROPORTIONAL REASONING

DIRECT RATIO Mr S Ab T Problem

INDIRECT RATIO Balance Beam Problem

FORMAL OPERATIONS

TRANSITION

CONCRETE OPERATIONS

TRANSITION

(12) QUANTITATIVE REASONING: Demonstrates that one ratio can be DIRECTLY or INDIRECTLY equivalent to a second ratio

(11) Systematically senates (f) one set of factors to achieve DIRECT equality with a second set of factors (Ft)

Systematically senates (f) one set of factors to achieve INDIRECT equality with a second set of factors (Ft)

(INRC) (f) INITIATES an action with one set of factors: I → disequilibrium

(N) NEGATES action taken in (f), returning factors to equilibrium: N → equilibrium

(R) forms RECIPROCAL of (f) by a second action of equal magnitude in opposite direction: I + R → equilibrium

(C) CANCELS effect of (R) by reversing/negating action (R): N + C → equilibrium

(10) Intuites DIRECT 1:1 or 1:MANY correspondence and seriation between two sets of factors

Intuites INDIRECT 1:1 or 1:MANY correspondence and seriation between two sets of factors

(9) QUALITATIVE REASONING: Intuites that two factors acting together may be equivalent to two other factors which act together

(8) Senates two factors in DIRECT 1:MANY correspondence

Senates two factors in INDIRECT 1:MANY correspondence

(7) Computes DIRECT correspondence for 1:MANY

Computes INDIRECT correspondence for 1:MANY

(6) QUALITATIVE REASONING: Intuites a 1:MANY correspondence that can be DIRECT or INDIRECT.

(5) Senates two factors in DIRECT 1:1 correspondence

Senates two factors in INDIRECT 1:1 correspondence

(INRC) (4) Senates (arranges in order) a single factor: 0-1-2-3-4-5...n or n...5-4-3-2-1-0

(f) Inserts additive action 0+1 = 1 1+1 = 2 2+1 = 3 ... n+1 = (n+1)

(N) Negates action in (f) 1-1 = 0 2-1 = 1 3-1 = 2 (n+1)-1 = n

(3) Computes DIRECT 1:1 correspondence

Computes INDIRECT 1:1 correspondence

(2) QUALITATIVE REASONING: Intuites that two factors can vary together (DIRECTLY) or in opposite directions (INDIRECTLY)

(1) Intuites DIRECT 1:1 correspondence

Intuites INDIRECT 1:1 correspondence

KEY: ——— Intuitive, qualitative hunch

——— Action followed by stated conclusion

RWP: 86

DISCUSSION

How can the mental processing model in Figures 3 and 4 promote both public and professional understanding of the challenges associated with teaching logical thinking? How can it promote discussion of issues related to this task as student populations become increasingly diverse in their prior knowledge and previous experience?

It is hoped that the mental processing model presented here will stimulate teachers and researchers to rethink the methods they use to teach proportional reasoning and to reevaluate the mental processes students exhibit in their effort to understand proportionality and ratio. The results of these efforts can, in a number of ways, ultimately lead to better public understanding of teachers' success in teaching proportional reasoning.

Mental processing models, if tested, refined and validated, serve as useful diagnostic and instructional tools. The model presented in Figures 3 and 4 is based on careful research but it has not been validated yet. Thus, experimentation by teachers and researchers using the model with other groups of students can lead to the revision or validation of the model. Such validation is certainly necessary before the model can be used predictably either for diagnostic or instructional purposes; but once that process takes place, a much wider impact can be expected from the use of the model in teaching proportional reasoning.

Within the mental processing model presented here, there appear subskills which are hypothesized as filling a gap in understanding the development of proportional reasoning. The existence and necessity of these skills (i.e., the ability to recognize 1:MANY correspondences, and the ability to serial order 1:MANY correspondences, thus forming seriations of ratios) need to be verified. Moreover, it is conceivable that additional essential subskills are yet to be discovered by teachers and researchers.

The question of teaching methods inevitably comes up when teachers and researchers discuss the development of proportional reasoning. Neither the mental processing model nor this paper have really addressed the efficiency of one teaching method over another (except for a brief reference). In the past, many teachers have relied upon a combination of lecture-discussion-chalkboard-homework methods to teach proportional reasoning in grades 7 through 12, and resorted to hands-on labs and demonstrations only when most of their students have failed to understand proportionality. The availability of a valid mental processing model for proportional reasoning could promote a more systematic search for evidence of when (under what conditions) one method of teaching this content is more effective than another.

Finally, there are a number of issues that face both the general audience and the teacher-researcher audience on the topic of teaching proportional reasoning. Foremost among the issues are our expectations for student acquisition of this important logical reasoning process, and the allocation of resources to meet the expectation. Other countries in the world have placed a high priority on students' mastery of proportional reasoning (see Table 1). England and China are two examples. The question must be asked whether the U.S. sets high enough expectations for its

students' mastery of proportional reasoning. It is disturbing to know that less than 20% of the 11th and 12th grade students from urban low income families are likely to have developed a reasoning skill that would allow them to make wise decisions about the use of their scarce resources (e.g., comparative shopping in a supermarket). It is also unsettling to realize that, along with these low income students, more than half of the students from middle income families may fail to develop this reasoning skill that is essential for understanding major concepts in science and mathematics.

The failure to achieve higher levels of student mastery has had to be tolerated since no sure-fire remedy was available. But can countries really afford such failures in the late twentieth century? Is it a question of expectations for student mastery or a question of appropriating enough resources to understand the problem and solve it?

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